UNIVERSITY OF CALICUT
SCHOOL OF DISTANCE EDUCATION

COMPLEMENTARY COURSE
   For
B.Sc. COUNSELLING PSYCHOLOGY

I SEMETER

MODULE 1: INTRODUCTION TO STATISTICS

MODULE 2: SAMPLING

MODULE 3: PROBABILITY AND EXPECTED VALUE

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MODULE - 1

Introduction to statistics.

Objectives: 1. To understand the meaning of statistics.
2. To know the need and importance of statistics in Psychology.

Meaning of statistics.

The word ‘Statistics’ is said to be derived from the Latin word ‘status’ or Italian word ‘statista’ or the German word ‘statistik’ each of which means a political state. In the olden days statistics was regarded as the ‘science of state craft’ related to the administrative activity of the state.

In general, ‘statistics’ conveys a variety of meaning to people. Some of them are:

- Statistics refers to numerical facts or data. (Data of birth date, school attendance, dropouts, employment market, agricultural products, etc are collected by statistical departments or other agencies which are known as statistics)
- Statistics refers to a method of dealing with quantitative information. (As a subject of study, it is a body of method of obtaining and analyzing data in order to make decisions on them.)

Thus statistics can be considered either as quantitative information or as a method of dealing with quantitative information.

Definitions of ‘Statistics’

Definitions of the term ‘statistics’ can be classified into two on the basis of how they see ‘statistics’. Some definitions consider it as statistical data (plural sense) where as some others as statistical methods (Singular sense).

Webster’s College dictionary defines statistics as facts or data of a numerical kind, assembled, classified, and tabulated so as to present significant information about a given subject (Plural).

Statistics as a branch of Mathematics can be defined as the science of collection, organization, presentation, analysis and interpretation of numerical data (Singular).

Based on this definition which is a more accepted one, there are five stages in a statistical investigation. They are

- Collection: The first step of any statistical investigation is collection of data which is the foundation of statistical analysis. If data are faulty, the conclusions drawn can never be reliable. So utmost care must be taken while collecting data. The data collected may be primary (collected by the investigator) or secondary (available from existing published or unpublished sources).
- **Organization:** A large mass of figures that are collected needs organization, the first step of which is editing. During editing, any omission, inconstancies irrelevant and wrong computation are adjusted or corrected.

The second step in organization is classification in which data is arranged according to some common characteristics. The third step in organization is tabulation. In tabulation, data is arranged in rows and columns so that there is absolute clarity in the data presented.

- **Presentation:** Presentation of data facilitates statistical analysis. Usually, the organized data is presented either using diagrams or graphs.

- **Analysis:** The data can be analyzed using different methods ranging from simple observation of the data to complicated, sophisticated and highly mathematical techniques. Commonly used methods of statistical analysis are measures of central tendency, measures of dispersion, correlation, regression etc.

- **Interpretation:** Interpretation means drawing conclusion from the data collected and analyzed. Correct interpretation will lead to valid conclusions helping one to take suitable decisions.

As statistical methods help in taking decisions, a better definition is “statistics is a method of decision making in the face of uncertainty on the basis of numerical data and calculated risks”.

**Need and Importance of statistics in Psychology**

Statistics is widely employed as a tool and is a highly valuable one in the analysis of problems in nature, physical and social sciences. Statistical methods are indispensable in forecasting, controlling and exploring various phenomena.

Statistics helps in systematic collection, organization, presentation of numerical data and in drawing conclusions or making predictions on the basis of particular evidences.

In psychology, statistics is very much essential as it involves measurement of behaviours, especially in various experiments for theory formation and verification.

The use of statistics in the field of psychology can not be listed exhaustively, but some applications are given below.

1. **In measurement and evaluation.**

   Instruments to measure variables of psychological interests are to be developed and standardized to make the process of measurement and evaluation more accurate. Statistics is used in the process of construction and standardization of such psychological tools. The data collected through these tools are organized, analyzed and arrived at conclusion using the methods of statistics.

2. **In carrying out activities related to profession as a psychologist.**

   A psychologist has to take decisions regarding guidance or counseling based on data collected through surveys or experiments. He has to use statistical methods in conducting experiments or surveys and arriving at valuable conclusions.
3. In carrying out researches.

Research is needed for the expansion and verification of knowledge in any field. A researcher needs statistics for successful completion of his research. According to Guilford (1973), statistical methods help a researcher in

- Describing the work in an exact form.
- Making the procedure definite and exact.
- Summarizing the results in a meaningful and convenient format,
- Drawing general conclusions.
- Predicting and
- Analyzing the causal factors underlying complex and otherwise bewildering events.

4. In understanding and using the products of research

Statistics help the practitioners to be conversant with latest developments in the field. The interpretation and proper use of the results of applied and theoretical researches become possible with proper knowledge of statistics.

Thus statistics has much application in psychology and is highly useful for professionals in the field to discharge their duties effectively, to update their knowledge and to contribute significantly for the welfare of the society.

Pre-requisites of studying statistics

Statistics involves numbers and formulae and hence need fundamentals of mathematical computations and reasoning for its study. But one need not be a scholar in mathematics for learning the procedure of using statistical methods in dealing with data related to behavioural sciences. Basic knowledge of the four fundamental operations-addition, subtraction, multiplication and division of rational and irrational numbers, square and square roots of numbers, proportion and percentage and simplification of complex expressions are needed for applying statistics in psychology. Besides the essential mathematical knowledge, the nature of variable, types of measurement, various statistical symbols and theory of probability are required for understanding statistical methods.

What one need to be successful in use of statistics are systematic study of the subject and an analytic mind to arrive at accurate conclusions.

Misuses of statistics

There is a general comment that ‘statistics can prove anything’. The main reasons for this comment are -

i) Figures are convincing and hence people are easily led to believe them.

ii) Data can be manipulated in such a manner as to establish foregone conclusions.

iii) Even if correct figures are used, they may be presented in such a manner that the reader is misled.

It is to be remembered that ‘statistics neither proves anything nor disproves anything’. It is only a tool –a method of approach. Tools, if properly used, give accurate results, if misused, prove disastrous.
MODULE-2

SAMPLING

Objectives: 1. To understand the concept of population and sampling.

2. To know the importance of sampling.

3. To understand various types of sampling methods.

4. To know about sampling error and non sampling error.

The data for statistical analysis may be primary or secondary. When secondary data is not available for the problem under study, primary data is to be collected using appropriate method and tool from the concerned group. This can be done either by census method or sampling method.

In census method data is collected from each member (unit) of the population. A population is the complete set of items which are of interest in any particular situation. For example, if we want to study the general anxiety of adults in Kerala, each and every adult of the state constitute the population and in census method, data is collected from each adult who is a resident of Kerala.

Here, no unit is left out and hence greater accuracy may be ensured; But when we consider the effort, money and time required for carrying out the data collection, this method is not economical. If the population is infinite or vary large, census method becomes difficult or impractical.

To rectify these drawbacks, sampling technique may be used in which only a part of the population (universe) is studied and conclusions are drawn on that basis for the entire population. That is, a sample is studied in order to draw inferences about the population.

Merits of sampling.

- Economical. In a sample survey, investigator collects data from a small representative group and hence it is economical with respect to time, effort and money.

- More reliable results. The result obtained through sample survey will be more reliable, accurate and complete. The extend of error due to sampling can be determined.

- More detailed information. It is possible to collect more details from the sample elements as sampling saves time and money.

- In some cases it will be impossible to collect data from the entire population, especially when data collection procedure is destructive in nature like in testing the durability of bulbs.

Demerits

Some demerits of sampling are:

- If there occurs error in sampling, the results may be inaccurate and misleading.

- Sampling to be perfect need service of experts in the field.
Complicated sampling design makes the study more laborious and expensive than a census.

If data regarding each and every unit of the population is needed, sampling method cannot be followed.

If the population is small, sampling method is not appropriate.

**Methods of sampling.**

Sampling methods can be classified into two

- Probability sampling and
- Non-probability sampling.

Probability sampling includes sampling process applying the laws of probability. If the laws of probability are not included in the procedure of sample selection, and is based on personal judgment of one or other type, it is non-probability sampling.

**Major methods under probability sampling are**

- Simple random sampling.
- Stratified sampling.
- Cluster sampling and
- Systematic sampling.

**Simple random sampling**

In simple random sampling, each and every item of the population is given an equal chance of being included in the sample. To ensure randomness of the sample, any one of the following methods can be adopted.

i) Lottery method: In this method all items of the universe are numbered on separate slips of identical size and shape. After folding, blind folded selection is made of the number of slips required to constitute the desired sample size.

ii) Table of Random Numbers

The procedure of sample selection will be simplified if Table of random numbers is used. In this method, units of the population are numbered continuously and using the table of random numbers sample elements are selected.

There are different types of table of random numbers, more popular ones are a) Tippett’s table of random numbers b) Fisher and Yate’s numbers and c) Kendall and Babington Smith numbers. These tables list numbers randomly and one can start at any point and count the required number of items. The units with this selected numbers (or with digits in the last places if population size is small) are included in the sample.

**Merits of random sampling**

1. Personal bias in sample selection which may affect the results can be completely avoided.
2. A random sample will represent the population more accurately than non-probability sampling methods. As the size of the sample increases, the sample becomes increasingly representative of the population.

3. The accuracy of estimation can be determined by the researcher.

**Demerits**

1. Random sampling necessitates a completely catalogued population.
2. Usually, large samples are needed in random sampling to get a reliable result.
3. In field surveys, the widely dispersed cases may increase the time and cost of data collection.
4. Sample selected through random sampling need not be a true representative one, especially when the population contains significant subgroups like male/female, rural/urban etc.

2. **Stratified sampling.**

In this method, the population is divided into various strata or parts and a sample is drawn from each stratum at random. If sample elements drawn from each stratum is proportional to their occurrence in the universe, it is proportional stratified sampling, and if equal number of cases is taken from each stratum regardless of how the stratum is represented in the population it is disproportional stratified sampling.

**Merits**

1. The sample selected will be more representative as the population is first divided into various strata and then a sample is drawn from each stratum.
2. Stratified sampling ensures greater accuracy.
3. Compared with random sampling, stratified method ensures more geographically concentrated sample elements.

**Demerits**

1. Utmost care must be exercised in dividing the population into various strata. Each stratum must contain homogeneous items.
2. Random selection of items from strata is difficult.

3. **Systematic sampling.**

This method is popularly used in those cases where a complete list of the population from which sample is to be drawn is available. A systematic sample is formed by selecting one unit at random and then selecting additional units at evenly spaced intervals until the sample has been formed. Here first item is selected randomly, generally following lottery method. Subsequent items by taking every \( k \text{th} \) item from the list where ‘k’ refers to the sampling interval (sampling ratio)
$k = \frac{N}{n}$ where $N$- Population size, $n$- Sample size.

**Merits:**

1. Systematic sampling is simple and convenient.

2. It is economical.

3. The results obtained from systematic samples are generally satisfactory and if the population is sufficiently large, the results from a systematic sample will be similar to those obtained by proportionate stratified sample

**Demerits**

1. If the population has hidden periodicities, the sample drawn through systematic sampling will not be a true representation of the population.

2. If the population is ordered in a systematic way with respect to the characteristics under study, then also there is a risk of poor representation of the sample.

4. **Cluster sampling (Multi-stage sampling)**

   Under this method, random selection is made of primary, intermediate and final units from a given population. There are several stages in which the sampling process is carried out. Initially random samples of first stage units are selected. As the second step, from the first stage units, second stage units are selected randomly. Further stages are added as required.

   **Illustration:** If we want a sample of 5000 secondary school students of Kerala State, we can take the educational districts of Kerala as the first units. After selecting required number of districts (say seven) randomly, sub districts of each district constitute the second units. Required number of sub districts can be randomly selected (say 20). Secondary schools constitute the third level units and some schools are selected randomly from the sub districts selected. From these schools, standards can be randomly selected and finally the classes are selected which constitute the final sample.

**Merits:**

1. There is flexibility in sampling. It enables existing divisions and sub-divisions of the population to be used as units at various stages and a large area can be covered with more accuracy.

2. If the population is infinite or very large i.e. when it is difficult to list all the population units, cluster sampling can be used.

**Demerits:**

A multi-stage sample is in general less accurate than a sample containing the same number of final stage units selected through other methods.
Major non-probability sampling methods are:

1. Judgment (Purposive)sampling.
2. Quota sampling and
3. Convenience sampling.


   In this method, the choice of sample items depends exclusively on the judgment of the investigator. The investigator includes those items in the sample, which he thinks are most typical of the universe/population with regard to the characteristics under consideration.

   **Merits**
   
   1. When the population contains only a small number of sampling units, random selection may miss more important elements, where as judgment sample would certainly include them in the sample.

   2. In solving everyday business problems and making public policy decisions, executives and public officials are pressed for time and cannot wait for probability sampling designs. Then judgment sampling is the only practical solution.

   **Demerits**
   
   1. This method is not scientific. It may be influenced by the personal bias of the investigator.

   2. If the investigator has enough knowledge about the population and has good judgment, the resulting sample will be representative of the population; otherwise, the inferences based on the sample will be erroneous.

   3. There is no objective method to determine the size of sampling error.

2. Quota sampling.

   In quota sampling, quotas are set up according to some specified characteristics such as so many in each of several income groups, so many in each age etc. Each interviewer is asked to interview a certain number of persons which constitute his quota. Within the quotas, the selection of sample items depends on personal judgment.

   **Merits**
   
   1. Quota sampling is used in public opinion studies.

   2. It may provide satisfactory results if the interviewers are trained and follow the instructions closely.

   **Demerits**
   
   1. There is possibility of personal bias which may affect the results.

   2. The sample may not be a true representation of the population.
3. Convenience Sampling.

A convenience sample or accidental sample is obtained by selecting ‘convenient’ population units. Here the sample elements are selected neither by probability method nor judgment but by convenience.

**Merits:** Convenience sampling can be used in pilot studies.

**Demerits:** There is a high probability of bias and will not be a representation of the population.

**Sampling error**

The results derived from a sample study may not be exactly equal to the true value in the population. That is, sampling gives rise to certain errors known as sampling errors (or sampling fluctuations). Sampling error may be biased or unbiased. Biased errors arise from any bias in selection, estimation etc. Unbiased errors arise due to chance differences between the members of population included in the sample and those not included.

**Non-sampling errors:**

When a complete enumeration of units in the universe is made, one would expect that it would give rise to data free from errors. But it is difficult to completely avoid errors of observation or ascertainment while processing data, tabulation errors may be committed affecting the final results. Errors arising in this manner are termed as non-sampling errors.

Thus, the data obtained in an investigation by complete enumeration, although, free from sampling error, would still be subject to non-sampling errors where as the results of a sample survey would be subject to sampling error as well as non-sampling error.
Module -3

Probability and expected value.

Objectives: 1. To know the meaning and definition of probability.

2. To develop understanding about the theorems of probability.

The word probability is very much used in general conversation, but the theory of probability has its origin in the games of chance related to gambling such as throwing a die, tossing a coin, drawing cards from a pack of cards etc. Girolamo Cardano (1501 – 76) an Italian Mathematician is considered as the first man who wrote a book on probability. An attempt to quantify probability was made by the Italian Mathematician Galileo (1564 -1642). A systematic and scientific foundation of mathematical theory of probability was laid in mid-seventeenth century by the French mathematicians Pascal (1623 -62) and Pierre De-Fermat (1601 – 65). Jacques Bernoulli (1654 -1705) through his treatise on probability has contributed much to the theory of probability. ‘The Doctrine of Chances’, the work of De Movire (1667 – 1754) was a remarkable step in the field of probability theory. Works of Thomas Bayes (1702 -61) and Pierre Simon Laplace (1749 -1827) gave a classical approach to probability. R A Fisher (1890-1962 ) and R V Mises (1883-1953) introduced empirical approach to probability through the concept of sample space. Modern approach to probability was given by Chebychev (1821 – 94), A. Markov (1856 -1922) etc.

Thus starting with games of chance, probability has become one of the tools of statistics. A basic knowledge of probability is essential to deal with statistics.

Definition of probability

The probability of a given event is an expression of likelihood or chance of occurrence of an event. (An event means the outcome of an experiment). The concept of probability of an event is explained differently by different schools of thought.

Classical (a priori) probability

The classical approach to probability is the oldest and simplest one. The basic assumption of classical theory is that the outcomes of a random experiment are ‘equally likely’.

Laplace defined probability as “the ratio of the number of ‘favourable’ cases to the total number of equally likely cases”. According to this definition probability of an event A

\[
P (A) = \frac{\text{Number of favourable cases}}{\text{Total number of equally likely cases}}
\]
Limitations

1. Not applicable when a simple enumeration of equally likely cases is not possible (e.g. Possibility of rain (rain or no rain) is not always equally likely).

2. Real life situations are often unlikely and disorderly and hence it will be difficult to apply classical probability concept.

2. Relative Frequency theory of Probability (empirical)

In this approach, probability of an event is the limit of the relative frequency as the number of observation increases indefinitely. If an event occurs ‘a’ times out of ‘n’, its relative frequency is ‘a/n’; the value which is approached by ‘a/n’ when n becomes infinity is called the limit of the relative frequency.

That is, \( P(A) = \lim_{n \to \infty} \frac{a}{n} \)

Limitations: Though, this approach is useful in practice, has difficulties from a mathematical point of view since an actual limiting number may not really exist.

3. Subjective approach to probability

The subjective probability is defined as the probability assigned to an event by an individual based on whatever evidence is available. Here probabilities are based on the beliefs of the person making the probability statement. For example, if a teacher is finding the probability of a particular student to be highest in the board examination, he may assign a value between zero and one according to his belief for possible occurrence. His belief will be based on the factors like past academic performance of the student, view of colleagues, attendance record etc.

Limitations

The personalistic approach being very broad and highly flexible, permits probability assignment to events for which there may be no objective data or for which there may be a combination of objectives and subjective data, and is very useful in business-decision making. But if at most care is not taken, the decisions made may be misleading.

4. Axiomatic Approach to probability

The Russian mathematician A.N. Kolmogorov introduced the axiomatic approach to probability in 1933 which introduces probability as a set function. There is no precise definition of probability in this approach but certain axioms/postulates are given on which probability can be calculated.

The whole field of probability theory for finite sample spaces (the set of all possible outcomes of a random experiment is called sample space of the experiment denoted as S) is based on the following three axioms.
1. The probability of an event ranges from zero to one. If the event cannot occur the probability will be zero and if it is certain, the probability is one.

2. The probability of the entire sample space is 1, i.e. \( P(S) = 1 \).

3. If A and B are mutually exclusive (disjoint) events then the probability of occurrence of either A or B, \( P(A \cup B) = P(A) + P(B) \).

**Calculation of Probability**

There are certain terms which are essential for an understanding of the calculation of probability. They are

1. **Experiment and events.**

   An act, which can be repeated under some given conditions, is known as an experiment. A random experiment is one whose results depend on chance.

   **Example:** Tossing of a coin, throwing of dice etc.

   The results of a random experiment are called outcomes. In tossing of a coin, the outcomes are Head (H) or Tail (T) while in the case of dice, the outcomes are 1, 2, 3, 4, 5 or 6. In these experiments all the possible outcomes are known in advance, and none of the outcomes can be predicted with certainty. While tossing a coin, one knows that possible outcomes are H and T, but no one can be certain that the outcome of a particular toss is H or T. These possible outcomes of a random experiment are called events.

   An event whose occurrence is inevitable when a certain random experiment is performed is called a certain or sure event and that can never occur is called an impossible event.

   In the experiment ‘Tossing of a coin’ the outcomes H and T are sure events where as coin showing not H or T, i.e. with edge as upward can be considered as an impossible event.

   An event which may or may not occur while performing a certain random experiment is known as a random event. Occurrence of H is a random event in the above random experiment of Tossing of a coin.

2. **Mutually Exclusive Events**

   Two events are said to be mutually exclusive (in compatible) if both cannot happen simultaneously in a single trial or the occurrence of any one of them precludes the occurrence of the other.

   **Eg.** When a coin is tossed, the occurrence of Head (H) precludes the occurrence of Tail (T). Both of them can not occur at the same time in a single toss. Hence these two events are mutually exclusive.

   If A and B are two mutually exclusive events Probability of occurrence of both of A and B,

   \[ P(AB) = 0 \]

   Three events A, B, C are mutually exclusive only if either A or B or C can occur.
3. **Independent and Dependent Events.**

Two or more events are independent when the outcome of one does not affect and is not affected by another.

Eg. If a coin is tossed twice, the outcome of first toss is not affecting or affected by the outcome of the second. That is, suppose in the first toss, a ‘Tail’ is obtained, then the second toss may result either T or H not influenced by the first outcome T. Similarly, the first outcome is not influenced by the outcome of the second.

Dependent events are those in which the occurrence or non-occurrence of one event in any one trial affects the probability of other events in other trials.

If a card is drawn from a pack of playing cards and is not replaced, this will alter the probability that the second card drawn is, say an ace.

4. **Equally likely events.**

Events are mutually likely when one does not occur more often than the others.

Example: If an unbiased coin is tossed, Head and Tail may be expected to be observed approximately the same number of times in the long run. But if the coin is biased, one should not expect each face to appear exactly the same number of times.

5. **Simple and Compound events.**

In simple event we consider the probability of the happening or not happening of single events.

In the case of finding the probability of drawing a white ball from a set of 20 black balls and 15 white balls, the event is simple as only one outcome is expected.

In compound events, the joint occurrences of two or more events are considered.

If two successive draws of 2 balls are made from a set of 20 black balls and 15 white balls, the probability of getting 2 white balls in the first draw and 2 black balls in the second draw - in this case we are dealing with a compound event as there are two single events occurring at the same time.

6. **Exhaustive events:**

Events are said to be exhaustive, if their totality includes all the possible outcomes of a random experiment.

Eg: In the case of throwing a dice, the possible events are showing the face with number 1, 2, 3, 4, 5 or 6. Thus the exhaustive number of cases is 6.
7. Complementary events

An event A is said to be the complementary event of B (and vice versa) if A and B are mutually exclusive and exhaustive.

E.g.: When a dice is thrown, occurrence of an odd number (A) and occurrence of an even number (B) are complementary events. Here A results in \{1, 3, 5\} and B in \{2, 4, 6\}. If these two are combined, the whole sample space is occurred and one prevents the occurrence of the other.

Theorems of Probability

Important theorems of probability are

1. The addition theorem
2. The multiplication theorem.

The addition theorem states that if two events A and B are mutually exclusive, the probability of the occurrence of either A or B is the sum of the individual probability of A and B.

I.e. If A and B are mutually exclusive or disjoint events \( P(A \cup B) = P(A) + P(B) \).

This theorem can be extended to more mutually exclusive events A, B & C

\[ P(A \cup B \cup C) = P(A) + P(B) + P(C) \]

**Proof:** If an event a can happen \( a_1 \) times and B \( a_2 \) times, and the total number of outcomes being ‘n’,

\[
P(A) = \frac{a_1}{n}, \quad P(B) = \frac{a_2}{n}
\]

Number of times either A or B can happen is \( a_1 + a_2 \). Thus probability of A or B

is \( P(A \cup B) = \frac{a_1 + a_2}{n} = \frac{a_1}{n} + \frac{a_2}{n} \)

I.e. \( P(A \cup B) = P(A) + P(B) \)

E.g.: One card is drawn from a standard pack of 52. What is the probability that the card selected is either a heart or a spade?

= The events that the card selected is a ‘heart’ and the card selected is a spade are mutually exclusive as only one card is selected and the card selected is ‘heart and spade’ is not possible.

\[
P(A) = \frac{13}{52}, \quad P(B) = \frac{13}{52}
\]

\[
P(A \cup B) = P(A) + P(B)
\]

\[
= \frac{13}{52} + \frac{13}{52} = 26 = 1
\]

\[
52 \quad 52 \quad 52 \quad 2
\]
If the events are not mutually exclusive, that is, if there is a probability of occurrence of both the events then \( P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(AB) \)

i.e. in the case of events which are not mutually exclusive, the probability of occurrence of A or B is the probability of A + probability of B – Probability of A and B.

E.g. What is the probability of selecting either a queen or a heart from a standard pack of cards.

The events that occurrence of a heart and the occurrence of a queen are not mutually exclusive as both can occur at a time. I.e. the card selected may be a queen of hearts. Thus

\[
P(A \text{ or } B) = P(\text{Queen or hearts})
\]

\[
= P(A) + P(B) - P(A \cap B)
\]

\[
=P(\text{Queen}) + P(\text{Hearts}) - P(\text{Queen of hearts})
\]

\[
= \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{17}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}
\]

This theorem can be extended as \( P(A \text{ or } B \text{ or } C) = P(A) + P(B) + P(C) - P(AB) - P(BC) - P(AC) + P(ABC) \)

**Multiplication theorem**

If two events A and B are independent, probability that they both will occur is the product of their individual probability. That is, \( P(A \text{ and } B) = P(A) \times P(B) \)

Extending this theorem to more events

\( P(ABC) = P(A) \times P(B) \times P(C) \).

**Proof:** If an event A can happen \( n_1 \) ways out of which \( a_1 \) are favourable for A and B can happen \( n_2 \) ways out of which \( a_2 \) are favourable for B, then a combination of successful event in both cases will be \( a_1 x a_2 \) from a total of \( n_1 \times n_2 \) cases.

Thus \( P(AB) = \frac{a_1}{n_1} \frac{a_2}{n_2} = \frac{a_1}{n_1} \times \frac{a_2}{n_2} = P(A) \times P(B) \)

**Conditional Probability and Bayes’ theorem**

If two events A and B are dependent (i.e. when B can occur only when A is known to have occurred or vice versa) then the conditional probability, probability of A given B
P (A/B) = \frac{P(AB)}{P(B)}

and probability of B given A

P (B/A) = \frac{P(AB)}{P(A)}

Proof: Suppose \( a_1 \) is the number of cases for the simultaneous happening of A and B out of \( a_1 + a_2 \) cases in which B can happen with or without happening of A

\[
P (A/B) = \frac{a_1}{a_1 + a_2} = \frac{a_1}{n} = \frac{P(AB)}{P(B)}
\]

The multiplication theorem can be also expressed as

\[
P (AB) = P (B) P (A/B)
\]

\[
P (AB) = P (A) \cdot P (B/A)
\]

\[
P (ABC) = P(A) \times P(B/A) \times P(C/AB)
\]

E.g. a bag contains 20 black and 15 white balls. Two balls are drawn at random one after the other without replacement. Find the probability that both balls drawn are black.

Let A be the event of getting a black in the first draw and B be that in the second draw

Then \[
P(A) = \frac{20}{35}
\]

\[
P (B/A) = P (\text{getting a black ball provided that black ball is taken in the first draw}) = \frac{19}{34}
\]

\[
P (A \text{ and } B) = P (\text{ Both balls are black}) = P (B/A) = \frac{19}{34} \times \frac{20}{35} = \frac{19 \times 7}{34 \times 4} = \frac{133}{136}
\]
This concept of conditional probability is used to predict the probability of one event on the basis of the information about the occurrence of another event. Using this conditional probability, Bayes’ theorem determines the probability that a particular effect was due to a specific cause.

This theorem is named after the British Mathematician Rev. Thomas Bayes (1702-61)

Bayes’ theorem can be expressed symbolically as

\[
P(A_i/B) = \frac{P(B/A_i) \cdot P(A_i)}{\sum P(B/A_j) \cdot P(A_j)}
\]

Where; A1, A2, A3…………..Ai………..An are n mutually exclusive and collectively exhaustive events.

B - is another event with \(P(B) \neq 0\)

For two mutually exclusive and exhaustive events A1 and A2, B another event which intersects events A1 and A2 (See Figure)

\[
P(A_1/B) = \frac{P(A_1 \cap B)}{P(B)}
\]

\[
P(A_2/B) = \frac{P(A_2 \cap B)}{P(B)}
\]

\[
P(B) = P(A_1 \text{ and } B) + P(A_2 \text{ and } B)
\]

\[
P(A_1 \text{ and } B) = P(B/A_1) \cdot P(A_1)
\]

\[
P(A_2 \text{ and } B) = P(B/A_2) \cdot P(A_2)
\]

\[
P(A_1/B) = \frac{P(B/A_1) \cdot P(A_1)}{P(B/A_1) \cdot P(A_1) + P(B/A_2) \cdot P(A_2)}
\]